Analysis of Environmental Factors which limit Plant Growth Using Fuzzy Modeling

Dmitry Kurtener¹, and Elena Krueger²

¹European Agrophysical Institute, Amriswil, Switzerland
²Insight Global, Greenwood Village, CO, USA

Correspondence: D. A. Kurtener, European Agrophysical Institute, Alpenstr. 1, 8580 Amriswil, Switzerland, e-mail: info@agrophysical.eu

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Abstract

Von Liebig formulated a concept in which plant growth is controlled by the supply of multiple essential resources. In this manuscript, several environmental factors which limit plant growth are evaluated using fuzzy modeling. Descriptions of several soil attributes which can limit plant growth (pH, phosphorus, potassium) are elaborated using fuzzy indicator models based on the trapezoid-shaped membership function. Also, fuzzy indicator models are developed for cases where soil temperature in the root area are lowered so that damage to winter crops occurs. The combined effect of several co-limitation factors is also evaluated using a weighted average operation. Several examples were discussed in this manuscript for illustration.

Keywords: fuzzy indicators, fuzzy set theory, plant growth limitation

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1. Introduction

It is well known that plants require multiple resources from their growing environments, such as nitrogen (N), phosphorous (P), water, light, temperature, and so on. Von Liebig formulated a concept where plant growth is controlled by the supply of multiple essential resources. According to Liebig’s law of the minimum, plant growth should be limited by the single resource (including environmental resources) which is in lowest supply relative to need (Gleeson & Tilman, 1992).

Studies of environmental factors that limit plant growth has been the subject of many publications. For example, resource limitation for plant growth was considered by Bloom et al. (1985) and interactions between plant nutrients, water, and carbon dioxide as factors limiting crop yields was considered by Gregory et al. (1997). Growth of plants in terrestrial ecosystems is often limited by the availability of nitrogen (N) or phosphorous (P) (Agren, 2012). In many of the papers it has been shown that biomass production of annual crops is often directly proportional to the amount of radiation intercepted, water transpired, and nutrients taken up. In many places the amount of rainfall during the period of rapid crop growth is less than the potential rate of evaporation, so that depletion of stored soil water is commonplace. The rate of mineralization of nitrogen (N) from organic matter and the processes of nutrient loss are closely related to the availability of soil water. Also, nutrient supply has a large effect on the quantity of radiation intercepted and hence, biomass production (Gregory et al., 1997).

Key interactions between nutrient limitation and climatic factors was studied by St.Clair et al. (2008). Mineral stress (nutrient deficiency and (or) ion toxicity) is a widespread phenomenon in forests around the world. However, with the exception of N limitation, its significance is often under appreciated. On weathered, acidic soils that support many of the world’s forests, P, Ca, and Mg deficiencies and toxicities of Al and Mn are important constraints to forest productivity (St.Clair et al., 2008). Also, Samejima et al. (2004) investigated root-shoot interaction as a limiting factor of biomass productivity. Bertrand et al. (1994) researched the effects of soil freezing and drought stress on abscisic acid content of sugar maple sap and leaves and root systems and root:mass ratio — carbon allocation under current and projected atmospheric conditions in arable crops were considered by Gregory et al. (1996). However, Davidson & Howarth (2007) noted that currently a mechanistic understanding of how co-limitation would function in these studied systems does not exist.

The multiple limitation hypothesis (MLH) has been developed as an alternative to Liebig’s law of the minimum. The multiple limitation hypothesis states that plants adjust their growth patterns such that they are limited by several resources simultaneously (Agren, 2012). More research is needed to evaluate MLH. One possible approach for evaluation of environmental factors, which limit the plant growth could be based on the use of the fuzzy set theory.

Fuzzy set theory is a mathematical approach that has successfully addressed many scientific and technical problems dealing with abstract questions. Recently, tools based on fuzzy sets theory and fuzzy logic have been developed for evaluation of different agricultural and environmental problems. Fuzzy sets are especially useful when insufficient data exist and it is impossible to characterize
uncertainty using standard statistical measures (e.g., mean, standard deviation, and distribution type) (May et al., 1997; Bogardi et al., 1996; Kurtener, & Yakushev, 2014). The fuzzy indicator models were applied to support spatial planning (Kurtener & Badenko, 2000, 2002), and for assessing agricultural lands for site-specific residue management (Kurtener & Badenko, 2000a). Also, fuzzy indicator models were successfully applied for evaluation of yield maps (Krueger et al., 2010a), evaluation of agricultural land suitability (Kurtener et al., 2008), assessment of soil quality (Torbert et al., 2008), evaluation of resources of agricultural lands (Torbert et al., 2009), and zoning of an agricultural field (Kurtener et al., 2011).

This manuscript uses fuzzy modeling to evaluate environmental factors that limit plant growth and is organized into two parts. The first part addresses the development of fuzzy indicator modeling to evaluate limitations due to environmental factor, and the second part contains examples which illustrates this approach.

2. Description of co-limitation factors using the fuzzy indicator models

The fuzzy indicator modeling can be used as a method for defining co-limitation factors. With the utilization of these models, it is possible to create “limitation fuzzy indicators” (LFIs). The use of LFIs provides a method for combining quantitative information (from experimental data) with qualitative information (opinions of experts). LFIs are defined as a numbers in the range from 0 to 1, which reflects an expert concept and is modeled by an appropriate membership function. LFI = 1 indicates that there is maximum limitation level on plant growth, while a LFI = 0 indicates that there is minimum limitation level on plant growth. Two simple examples of this model are illustrated with data from agricultural field experiments and meteorological observations.

Example 1

Crop yield controlled by soil attributes

In this example an empirical model formulated by Kaiumov (1977) was used. Kaiumov analyzed suitability of yield-controlled factors for crops and defined the intervals of soil attributes which are most suitable for crop growth (Table 1). In other words, according to Kaiumov’s empirical model, there exists an interval for a soil attribute so that if the values of this attribute lies within this interval then it is does not limit crop growth.

It should be noted that while the Kaiumov’s model may have considerable shortages, it serves well as an example to illustrate this suggested approach.

Modeling the limitation fuzzy indicators

In this case, the limitation fuzzy indicators (LFIs) can be described as:
Where \( mf(x) \) is the appropriate membership functions. In this study, we use the trapezoid-shaped membership function as follows:

Trapezoidal-shaped built-in membership function is characterized by four reference points: \( x_{\text{low1}} \), \( x_{\text{opt1}} \), \( x_{\text{opt2}} \) and \( x_{\text{low2}} \). These points are defined as follows:

- If \( x < x_{\text{low1}} \), then \( mf(x) = 0 \),
- If \( x_{\text{opt1}} < x < x_{\text{opt2}} \), then \( mf(x) = 1 \),
- If \( x > x_{\text{low2}} \), then \( mf(x) = 0 \).

Using data of Table 1, membership functions for three of the limitation fuzzy indicators were determined (figures 2 – 4). It should be noted that the choice of characteristics of membership function is somewhat arbitrary and should mirror an objective expert opinion. In our example, reference points are given in Table 2.

**Table 1. Interval values of soil attributes within which crop growth is most suitable (Kaiumov, 1977)**

<table>
<thead>
<tr>
<th>Soil</th>
<th>pH</th>
<th>SOM, %</th>
<th>( P_2O_5 ), mg kg(^{-1})</th>
<th>( K_2O ), mg kg(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loam</td>
<td>6.5 - 7</td>
<td>1.8 - 2.2</td>
<td>250 - 280</td>
<td>200 - 260</td>
</tr>
<tr>
<td>Loamy sand</td>
<td>6 – 6.5</td>
<td>2 – 2.4</td>
<td>200 - 250</td>
<td>180 - 200</td>
</tr>
<tr>
<td>Sandy</td>
<td>5.5 - 6</td>
<td>2.2 - 2.6</td>
<td>180 – 200</td>
<td>140 - 160</td>
</tr>
<tr>
<td>Turf</td>
<td>5 – 5.5</td>
<td>-</td>
<td>500 - 600</td>
<td>600 - 800</td>
</tr>
</tbody>
</table>

**Table 2. Characteristics of membership function**

<table>
<thead>
<tr>
<th>Membership function</th>
<th>( x_{\text{low1}} )</th>
<th>( x_{\text{opt1}} )</th>
<th>( x_{\text{opt2}} )</th>
<th>( x_{\text{low2}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>pH</td>
<td>5.5</td>
<td>6</td>
<td>6.5</td>
<td>7</td>
</tr>
<tr>
<td>( P_2O_5 ), mg kg(^{-1})</td>
<td>160</td>
<td>200</td>
<td>250</td>
<td>290</td>
</tr>
<tr>
<td>( K_2O ), mg kg(^{-1})</td>
<td>100</td>
<td>140</td>
<td>200</td>
<td>250</td>
</tr>
</tbody>
</table>
Fig 2. Membership function for description of pH.

Fig 3. Membership function for description of $P_2O_5$.

Fig 4. Membership function for description of $K_2O$. 
**Example 2**

**Frost damage on winter crops**

Moiseychik (1975) shows that frost often destroys winter crops. Several factors such as the soil temperature in the root area (at 2 cm depth), the depth of the boundary between the frozen and melted soil layers, the thickness of the snow cover, the kind of crops, the crop’s resistance to frost, and the density of the planting (the number of stems) determine the amount of thermal damage to winter crops. Analysis of the empirical data by Moiseychik showed two intervals could be identify using soil temperature in the root area \(T_{\text{root}}\) as an indicator. In the first interval the thermal constraints for growth of winter crops are absent, while in the second interval the thermal conditions for winter crops are destructive to plant growth. Between these intervals there is a transition area (Yakushev et al., 2002).

<table>
<thead>
<tr>
<th>Winter crops</th>
<th>Characteristics of winter crops</th>
<th>Interval of suitable (T_{\text{root}})</th>
<th>Interval of inadmissible (T_{\text{root}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>barley crops</td>
<td>an underdeveloped crop</td>
<td>(-7 &lt; T_{\text{root}} = -12)</td>
<td>(T_{\text{root}} &lt; -12)</td>
</tr>
<tr>
<td></td>
<td>a normal crop</td>
<td>(-14.8 &lt; T_{\text{root}} = -19.2)</td>
<td>(T_{\text{root}} &lt; -19.2)</td>
</tr>
<tr>
<td></td>
<td>an outgrowing crop</td>
<td>(-7 &lt; T_{\text{root}} = -13)</td>
<td>(T_{\text{root}} &lt; -13)</td>
</tr>
<tr>
<td>wheat crops</td>
<td>an underdeveloped crop</td>
<td>(-11 &lt; T_{\text{root}} = -17)</td>
<td>(T_{\text{root}} &lt; -17)</td>
</tr>
<tr>
<td></td>
<td>a normal crop</td>
<td>(-14 &lt; T_{\text{root}} = -19.4)</td>
<td>(T_{\text{root}} &lt; -19.4)</td>
</tr>
<tr>
<td></td>
<td>an outgrowing crop</td>
<td>(-10 &lt; T_{\text{root}} = -15)</td>
<td>(T_{\text{root}} &lt; -15)</td>
</tr>
<tr>
<td>rye crops</td>
<td>an underdeveloped crop</td>
<td>(-11 &lt; T_{\text{root}} = -22)</td>
<td>(T_{\text{root}} &lt; -22)</td>
</tr>
<tr>
<td></td>
<td>a normal crop</td>
<td>(-14 &lt; T_{\text{root}} = -25)</td>
<td>(T_{\text{root}} &lt; -25)</td>
</tr>
<tr>
<td></td>
<td>an outgrowing crop</td>
<td>(-11 &lt; T_{\text{root}} = -22)</td>
<td>(T_{\text{root}} &lt; -22)</td>
</tr>
</tbody>
</table>

**Modeling the limitation fuzzy indicators**

Using this information, the limitation fuzzy indicators (LFIs) can be described as:

\[
\text{LFI} = mf(x),
\]
where \( mf(x) \) is the appropriate membership functions, which in this study was the sigmoidal-shaped membership function.

Membership functions for winter barley (for an underdeveloped crop, a normal crop, and an outgrowing crop) were elaborated in figure 5. Index of soil temperature in the root area was defined as follows:

\[
\text{Index} = A + T_{\text{root}}
\]

where \( A \) is equal:

- 15 for an underdeveloped crop
- 25 for a normal crop
- 15 for an outgrowing crop

Thus, \( LFI \) describes the thermal conditions in the root area of winter barley where soil temperatures move from the values suitable for crop growth to values that will cause damage.

3. Assessment of combined effect of several factors using the fuzzy aggregation algorithms

Assessment of combined effect of several co-limitation factors can be made with the use of fuzzy aggregation algorithms. In this study, weighted average was utilized. Specifically, combined fuzzy indicators (CFI) was defined as follows:

\[
CFI = \sum w_i LFI(x_i),
\]

where \( w_i \) is a weighted coefficient used for assessment of the significance of the \( x_i \) co-limitation factor, \( \sum w_i = 1 \), and \( 0 < w_i < 1 \). \( LFI(x_i) \) denotes a fuzzy indicator for \( x_i \).

For illustration of these procedures, 4 expert assessments (Table 4) situations were considered. Results of modeling the combined fuzzy limitation indicators (CFI) calculated from the different assessments on the significance of the limitation factors are pretend in Table 5.

<table>
<thead>
<tr>
<th>Table 4. Variants of assessment of significance of the limitation factors</th>
<th>Expert assessment of the weighted coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variant of assessment of significance of the limitation factors</td>
<td>pH</td>
</tr>
<tr>
<td>1 (equivalent)</td>
<td>0.33</td>
</tr>
<tr>
<td>2 (( P_2O_5 ) is more significance than other)</td>
<td>0.15</td>
</tr>
<tr>
<td>3 (pH is more significance than other at large value)</td>
<td>0.7</td>
</tr>
<tr>
<td>4 (( K_2O ) is more significance than other)</td>
<td>0.15</td>
</tr>
</tbody>
</table>
Fig 5. Memberships function for description of thermal conditions in the root area of winter barley when soil temperature transits from the values suitable for crop growth to values that will cause damage (a - the underdeveloped crops, b - a normal crop, and c - an outgrowing crop).
Table 5. Results of modeling the individual fuzzy limitation indicators (LFIs) and the combined fuzzy limitation indicators (CFI) reflecting different assessment of the significance of the limitation factors

<table>
<thead>
<tr>
<th>LFI on pH</th>
<th>LFI on P₂O₅</th>
<th>LFI on K₂O</th>
<th>CFI reflected different points of view on significance of the limitation factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F1</td>
<td>F2</td>
<td>F3</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.375</td>
<td>0</td>
<td>0.453</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.33</td>
</tr>
<tr>
<td>0.3</td>
<td>1</td>
<td>0.4</td>
<td>0.561</td>
</tr>
<tr>
<td>0.2</td>
<td>1</td>
<td>0</td>
<td>0.396</td>
</tr>
<tr>
<td>0.7</td>
<td>0</td>
<td>1</td>
<td>0.561</td>
</tr>
<tr>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td>0.495</td>
</tr>
</tbody>
</table>

4. Conclusion

The fuzzy modelling procedures provide a method to combine structured information (quantitative experimental data) with blurred or ambiguous information (opinions of experts). Therefore, fuzzy indicator models are an appropriate method for definition of individual co-limitation factors (LFI). Co-limiting factors for two cases were described in this manuscript. They included a) description of soil conditions when soil characteristics (pH, P₂O₅, K₂O) transits from values suitable for crop growth to those that damage crop growth; and b) description of thermal conditions in the root area of winter crops when soil temperatures transits from the values suitable for crop growth to values that kill plants. The combined effect of several co-limitation factors (CFI) can be assessed with the use of LFIs. The application of fuzzy aggregation algorithms provided a method to take into account different points of view or assessment of the significance of these limiting factors. Several different situations were discussed.

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References


